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Electromagnetically induced tunnelling suppression in a flux qubit

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Abstract. Motivated by recent experiments wherein Josephson devices are irradiated by microwaves fields or are coupled to LC resonators, we theoretically investigate the dynamics of a flux qubit coupled to a monochromatic bosonic mode. We define strong coupling conditions under which the qubit tunnelling frequency between the localized flux states can be controlled and even suppressed. The practical realization of such a regime leading to this hindered dynamics is discussed.

PACS. 03.67-a Quantum information – 03.67.Lx Quantum computing – 85.25.Dq Superconducting quantum interference devices (SQUIDs)

In the past few years Josephson junctions-based devices have been widely studied both theoretically and experimentally as possible candidates for the implementation of a quantum computer [1–9]. In fact, under appropriate values of external bias pulses, they behave as two level systems which can be easily used as quantum bits. Several systems like ion traps and NMR systems [10,11], have been suggested as physical realizations of quantum bit but Josephson devices exhibit the main advantage of being scalable up to large numbers of qubits as nanocomponents embedded in an electronic circuit. Moreover, it is possible to prepare these devices in a prefixed initial state or in a superposition of states and to control their dynamics by external voltages and magnetic fluxes [12]. When the charging energy E_C overcomes the Josephson energy E_J, like in the case of Cooper pair boxes, the macroscopic degree of freedom used to store and process quantum information is the charge Q on the junction. On the contrary, large area current-biased Josephson junctions or superconducting quantum interference devices (SQUIDs), for which $E_J \gg E_C$, are describable in terms of the phase difference φ between the order parameters of the superconducting junction electrodes or in terms of the magnetic flux ϕ ($\propto \varphi$) threading the squid loop. Nakamura's group has shown that it is possible to control the coherent quantum state evolution of a Cooper pairs box by applying a short voltage pulse *via* a gate electrode [1]. Moreover they prove the existence of Rabi oscillations when this Josephson-junction charge two level system is driven by a strong oscillating field [2]. A similar experiment performed by Yang Yu *et al.* [4] brought to light the existence of microwave excited coherent Rabi oscillations between the macroscopic quantum states of a Josephson junction. In addition spectroscopic measurements demonstrate the existence of superpositions of fluxoid states and persistent-current states in superconducting quantum interference devices (SQUIDs) [13,14]. Finally coherent time evolution between its quantum states has been observed manipulating the qubit by resonant microwave pulses [8]. In all these experiments, very important in view of the practical realization of solid state quantum gates and chips, one of the crucial problem concerns the coherent control and manipulation of the state of the qubit *via* external parameters or *via* the coupling with another qubit or with the single mode of a quantized electromagnetic field of a resonant cavity. It is then clear why the dynamics of Josephson devices exposed to quantized electromagnetic fields has attracted a growing number of authors over the last few years [15–21]. In a previous paper [22], the dynamics of a flux qubit irradiated by a resonant quantized electromagnetic field has been investigated finding that some appropriate external parameters may be fixed in such a way to control the dynamical replay of the total system. It has been proved, for instance, that the flux qubit-single mode field may be forced to exhibit interesting features like the occurrence of an oscillatory appearance and disappearance of entanglement between the two subsystems or the possibility of generating maximally entangled superpositions of clockwise and counterclockwise supercurrents states in the loop.

In this paper we investigate the interaction between a flux qubit and a quantized electromagnetic field of a microwave structure assuming off-resonance conditions. We demonstrate that when the energy of the bare qubit is negligible with respect to both the energy of a single field

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excitation and the interaction energy, then the natural tendency of the bare qubit to tunnel between its two localized ground states may be progressively hindered. Our main result is that, appropriately choosing the state of the electromagnetic field and suitable values for some externally controllable parameters entering in the expression of the coupling term, the tunnelling of the qubit may be frozen. The importance of conceiving control techniques has been recently recognized by the group of Mooij [23]. They show, for example, that the strongly reduction (freezing) of the tunnelling Δ between the two qubit states before the measurement process starts, allows for much slower measurements, thus for a better estimation of decoherence rates. The paper is organized as follows. We start by introducing the Hamiltonian model characterizing the system under scrutiny. Then we define the strong coupling regime bringing to light interesting features of the matter-radiation system dynamics in this coupling limit and in particular discussing the electromagnetically tunnelling induced suppression phenomenon. Finally, we estimate the possibility of observing such a behavior taking into account the currently available experimental setups.

Let us begin by describing the total matter-radiation system under study, that is a flux qubit inductively coupled to a single mode of a resonant microwave cavity [17,22].

It is well known that the simplest tunable flux qubit may be practically realized by a double *rf*-SQUID, a superconducting loop of inductance L broken by a small *dc*-SQUID and with an external magnetic flux ϕ_x applied to the loop [12,24]. This device behaves as a regular *rf*-SQUID with a tunable critical current

$$I_C(\phi_c) = 2I_{C0}\left|\cos\left(\pi\frac{\phi_c}{\phi_0}\right)\right| \tag{1}$$

and then a controllable parameter $\beta_L = 2\pi L I_C(\phi_c)/\phi_0$. Here I_{C0} is the critical current of both the two Josephson junctions interrupting the *dc*-SQUID loop, $\phi_0 = h/2e$ is the flux quantum and ϕ_c a second magnetic bias flux applied to the smallest *dc*-SQUID ring.

Under the conditions $\phi_x = \phi_0/2$ and $\beta_L > 1$, the Josephson potential $U(\phi) = -E_J\cos(2\pi\phi/\phi_0) + (\phi - \phi_x)^2/2L$ is characterized by two symmetric lower well separated by a barrier of height V_b at $\phi_x = \phi_0/2$ [25].

At sufficiently low temperature the physics of the system may be described in terms of the ground states in the wells $|L\rangle$ and $|R\rangle$ corresponding to the clockwise and counterclockwise sense of rotation of the supercurrent in the loop.

In other words, the squid behaves as a two state system whose effective Hamiltonian, with respect to the shifted coordinate $\phi - \phi_0/2$ and in the computational basis of the energy eigenstates

$$|\mp\rangle = \frac{1}{\sqrt{2}}[|R\rangle \pm |L\rangle] \tag{2}$$

assumes the following form

$$H_S = -\frac{\hbar}{2}\Delta\,\sigma_z. \tag{3}$$

The energy separation $\hbar\Delta$ between the ground state $|-\rangle$ and the excited state $|+\rangle$ of the squid is defined in terms of the frequency of oscillations between the two flux states $|L\rangle$ and $|R\rangle$ whose analytical expression is given by

$$\Delta \simeq 6\frac{V_b}{\hbar}\exp\left(-8\frac{V_b}{\hbar\omega_0}\right) \tag{4}$$

in the limit $0 < (\beta_L - 1) \ll 1$. Here $\omega_0 \simeq \sqrt{2(\beta_L-1)}/\sqrt{LC}$ is the frequency of small oscillations in the wells and $V_b \simeq \frac{3\phi_0^2}{8\pi^2 L}\frac{[\beta_L(\phi_c)-1]^2}{\beta_L(\phi_c)}$ the barrier height [24].

This flux qubit is inductively coupled to the single bosonic mode of a quantized electromagnetic field with a contribution to the total Hamiltonian given by

$$H_{Int} = \frac{2k}{L}\,\phi\phi_F \tag{5}$$

where $\phi = \frac{\phi_0}{2}\,\sigma_x$ is the total magnetic flux threading the ring, $\phi_F = \sqrt{\frac{\hbar}{2\omega_F C_F}}\,(a + a^\dagger)$ the magnetic flux induced by the quantized electromagnetic field and k the dimensionless coupling constant. There are several proposals for the realization of this theoretical scheme. For example, we may place the qubit in a superconducting transmission line [26]. In this case the qubit displays a dynamic interaction with the linear electromagnetic wave propagating in the transmission line. Another possible realization of the coupling is a configuration wherein the superconducting qubit is coupled to a lumped L-C circuit [27,28].

In all these realizations the qubit-field Hamiltonian can be cast in the following form:

$$\begin{aligned} H &= H_F + H_S + H_{Int} \\ &= \hbar\omega_F\left(a^\dagger a + \frac{1}{2}\right) - \frac{\hbar}{2}\Delta\,\sigma_z + B\Delta(a + a^\dagger)\sigma_x \end{aligned} \tag{6}$$

where the first term in the last line describes the free-field Hamiltonian, while the interaction term depends on the constant

$$B = \frac{k}{L}\sqrt{\frac{\hbar}{2\omega_F C_F}}\frac{\phi_0}{\Delta} \tag{7}$$

which has the same dimension of $\hbar$ and is characterized by the coupling strength k and the system parameters [22]. This Hamiltonian can also be interpreted as that of a spin $S = \frac{1}{2}$ having a Larmor frequency Δ in an external magnetic field, transversely coupled to a harmonic oscillator of frequency ω_F.

According to the rules of Quantum Mechanics, in the ideal case (without dissipation and zero noise) the probabilities P_L and P_R of finding an isolated *rf*-SQUID in its states $|L\rangle$ and $|R\rangle$ respectively are sinusoidal oscillating functions characterized by the bare frequency Δ. As we already underlined the possibility of controlling this tunnelling frequency is of interest both from a fundamental and an applicative point of view [23].

Motivated by these perspectives, here we propose a scheme for the control of the tunnelling frequency of a flux qubit *via* its exposition to a single-mode quantized electromagnetic field. To this end it is useful to consider the case of strong coupling between the two subsystems corresponding to the conditions:

$$\hbar\Delta \ll \hbar\omega_F,\ B\Delta. \tag{8}$$

Such inequalities may be easily satisfied by appropriately choosing the parameters of the matter-radiation system. For example, considering a SQUID with $L \approx 200$ pH, $C \approx 0,1$ pF and $\beta_L = 1,1$, according to equation (4), we obtain $\Delta \sim 1.5 \times 10^9$ rad s$^{-1} \ll \omega_F$. This means that, if the field frequency $\omega_F \equiv 1/\sqrt{L_F C_F}$ is of the order of 10^{11} rad s^{-1}, the first condition $\Delta \ll \omega_F$ is well fulfilled. Moreover, since for these values of the system parameters C_F results of the order of 10^{-12} F, we find that also the second condition $\hbar \ll B$ is obeyed choosing currently realizable values of the constant k larger than 0.01 [24].

In such physical conditions, it is possible to treat the free SQUID Hamiltonian $-\frac{\hbar}{2}\Delta\ \sigma_z$ as a perturbation of the states of

$$H_0 = \hbar\omega_F\left(a^\dagger a + \frac{1}{2}\right) + B\Delta(a + a^\dagger)\sigma_x. \tag{9}$$

This approach to the study of Hamiltonian (6) is not new since has been considered by many authors in connection with rather different physical situations [29,30]. For example, Shore [31] devised an approximate self-consistent treatment of Hamiltonian (6) to derive the concept of reduced tunnelling frequency for paraelectric defects in simple ionic crystals. Moreover, the possibility of an instability in the ground state of this simple model of spin-phonon interaction in the strong coupling regime case has been investigated [32]. The new result reported in this paper is that a quantized electromagnetic field strongly coupled to a tunable *rf*-SQUID may influence in a remarkable way the dynamical behavior in its low-lying energy states subspace.

Since the magnetic flux threading the ring $\phi = \frac{\phi_0}{2}\sigma_x$ is a constant of motion ($[H_0, \phi] = 0$) we may classify the eigenstates of H_0 in terms its eigenvalues $\pm\frac{\phi_0}{2}$. In the first case the supercurrent flowing in the SQUID loop is counterclockwise ($\phi|R\rangle = +\frac{\phi_0}{2}|R\rangle$) in the second case is clockwise ($\phi|L\rangle = -\frac{\phi_0}{2}|L\rangle$). It is well known that it is possible to diagonalize Hamiltonian (9) by applying the unitary operator [33]

$$T_{R/L} = \exp\left[\pm\alpha(a - a^\dagger)\right] \tag{10}$$

where the symbol "R"("L") refers to the half the Hilbert space in which the expectation value of ϕ is $\frac{\phi_0}{2}$ ($-\frac{\phi_0}{2}$) and $\alpha = \frac{B\Delta}{\hbar\omega_F}$. The eigenstates originating by diagonalizing this Hamiltonian are given by

$$|\psi_n; R/L\rangle = T_{R/L}|R/L; n\rangle = |R/L\rangle e^{\pm\alpha(a-a^\dagger)}|n\rangle \tag{11}$$

where $|n\rangle$ are the field Fock states. The corresponding eigenvalues $E_n = \hbar\omega_F n - \frac{(B\Delta)^2}{\hbar\omega_F}$ are all twofold degenerate and are related to the two possible directions of supercurrent circulation in the loop.

What about the effect of the perturbation term $-\frac{\hbar}{2}\Delta\sigma_z$? This term connects the states with opposite expectation value of ϕ. In other words the not vanishing matrix elements are given by:

$$\langle\psi_n; R| - \frac{\hbar}{2}\Delta\sigma_z|\psi_m; L\rangle = \\ -\frac{\hbar}{2}\Delta\langle R|\sigma_z|L\rangle\langle n|e^{-2\alpha(a-a^\dagger)}|m\rangle = -\frac{\hbar}{2}\Delta A_{nm}. \tag{12}$$

In view of equation (8) it is possible to limit the perturbation calculations to order zero by considering the secular equations $E^2 - (\frac{\hbar}{2}\Delta A_{nn})^2 = 0$ whose eigenvalues are:

$$E_\pm = \pm\frac{\hbar}{2}\Delta|A_{nn}|. \tag{13}$$

Exploiting the well known result [34]

$$\langle n|e^{\,\alpha(a-a^\dagger)}|m\rangle = \sqrt{\frac{m!}{n!}}\ (\alpha)^{n-m}\ e^{-\frac{\alpha^2}{2}} L_n^{n-m}(\alpha^2) \tag{14}$$

with $n \geq m$ and $L_n^{n-m}(\alpha^2)$ associated Laguerre polynomials, finally yields:

$$A_{nn} = e^{-2\alpha^2} L_n^0\left(4\alpha^2\right). \tag{15}$$

In view of equation (13), we thus find that the two degenerate energy states given by equation (11), corresponding to the unperturbed eigenvalue E_n, are mixed and split.

The new eigenstates $|\Psi_{qn}\rangle$ (with $q \equiv 1, 2$) corresponding to the eigenvalues

$$E_{qn} = E_n + (-1)^q\frac{\hbar}{2}\Delta|A_{nn}| \tag{16}$$

are respectively given by

$$|\Psi_{qn}\rangle = \frac{1}{\sqrt{2}}[|\psi_n; L\rangle + (-1)^q|\psi_n; R\rangle]. \tag{17}$$

A relevant consequence of this result is that by preparing the total system in the state $|\psi_n; R\rangle$ after a suitable time the system will be in the state $|\psi_n; L\rangle$, the switching frequency being:

$$\Delta' = \frac{E_{2n} - E_{1n}}{\hbar} = \Delta|A_{nn}| = \Delta\left|e^{-2\alpha^2} L_n^0\left(4\alpha^2\right)\right|. \tag{18}$$

Remembering that $\alpha = \frac{B\Delta}{\hbar\omega_F}$ and in view of equation (7), this result clearly shows the non-linear dependence of the qubit frequency Δ' both on the number of (displaced) photons n in the field and on the coupling constant k. In other words equation (18) describes the possibility of controlling the tunnelling frequency between the two washboard-potential wells acting upon appropriate external parameters. In particular this equation forecasts the

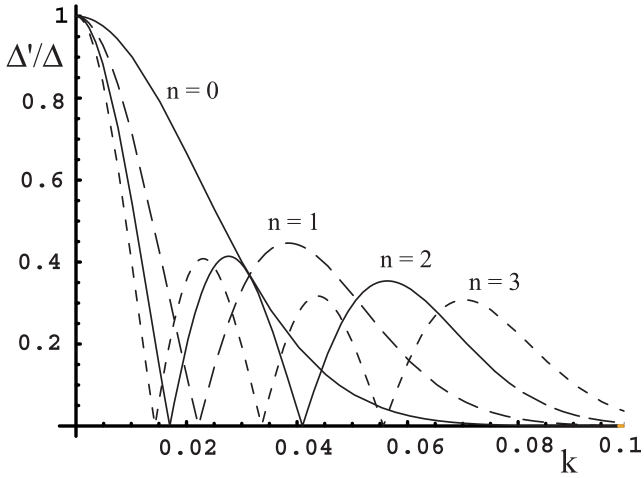

Fig. 1. Plot of the normalized tunnelling frequency Δ'/Δ with respect to k for different values of n.

possibility of making $\Delta' = 0$, that is to freeze the system in its initial condition. This fact may be illustrated by Figure 1 where the behavior of Δ'/Δ with respect to k for $n = 0, 1, 2$ and 3 is plotted.

In order to highlight the possibility to tune $\Delta' = 0$, we assume that the system is initially prepared in the state

$$|R; \alpha 1\rangle = |R\rangle e^{\alpha(a^\dagger - a)}|1\rangle \equiv |\psi_1; R\rangle \tag{19}$$

that is the qubit with a counterclockwise supercurrent in the loop and the quantized electromagnetic field prepared in the displaced number state $|\alpha 1\rangle = D(\alpha)|1\rangle$, where as before $\alpha = B\Delta/\hbar\omega_F$. This well known non-classical state may be easily realized by driving the quantized field of a single-mode resonant cavity prepared in its Fock state $|1\rangle$ by a classical current [34]. Expressing the time evolution of the initial state of the system in terms of the eigenstates $|\Psi_{1n}\rangle$ and $|\Psi_{2n}\rangle$ as

$$|\psi_1; R\rangle_t = \frac{1}{\sqrt{2}}\left[-|\Psi_{11}\rangle e^{-E_{11}t/\hbar} + |\Psi_{21}\rangle e^{-E_{21}t/\hbar}\right], \tag{20}$$

it is easy to demonstrate that the expectation value of the qubit flux operator $\phi = \frac{\phi_0}{2}\sigma_x$ is given by

$$\begin{aligned}\langle\phi\rangle_t &= \frac{\phi_0}{2}\cos[(E_{21} - E_{11})t/\hbar] = \\ &= \frac{\phi_0}{2}\cos[\Delta\exp(-2\alpha^2)L_1(4\alpha^2)t].\end{aligned} \tag{21}$$

This expression clearly shows that it is possible to suppress the tunnelling between the localized flux states $|L\rangle$ and $|R\rangle$ ($\langle\phi\rangle_t = \frac{\phi_0}{2}$, $\forall\ t$) if $4\alpha^2$ is equal to a zero of the Laguerre polynomial $L_1(z)$. Confining, in particular, to its first zero corresponds to take $\alpha = 0.5$ and then $k \equiv \bar{k} \simeq 0.022$ satisfactorily compatible with our definition of strong qubit-field regime. Of course, one cannot neglect that to perfectly tune k with this value nullifying $L_1(z)$ might be difficult to achieve from an experimental point of view. This qualitatively means that in such conditions there will still be some residual tunnelling between the two qubit states. More quantitatively, considering values of k in the range $[\bar{k} - \delta, \bar{k} + \delta]$, with $\delta = 0.0001$, we obtain $\Delta'/\Delta \approx 0.0055$ corresponding to a maximum period of the order of 0.8×10^7 s. Such a behavior, clearly traceable back to the sharp variation of $L_1(4\alpha^2)$ around $\alpha = 0.5$ against the coupling constant k on which α depends, requires a very accurate tuning of the parameter k to successfully hinder the qubit dynamics.

Our proposal might be of relevant interest in building up superconducting quantum computers. Of course, in view of the practical realization of theoretical schemes in the context of quantum computing, a crucial point is to estimate and how to reduce undesirable decoherence effects stemming from environment. We must indeed take into account the fact that a *rf*-SQUID is a much more complex device with respect to the ideal one previously described. In fact, the whole SQUID chip is made at least by a fully integrated Nb/AlO$_x$/Nb Josephson device, including the bias coil, coupled, for example *via* a gradiometric flux transformer, to a read-out system, typically based on a Josephson interferometer (a *dc*-SQUID). Moreover, in our case we must take into account the experimental setup related to the coupling between the SQUID and the source of the quantized electromagnetic field. In other words, the coupling with electromagnetic degrees of freedom, which, as shown, is a fundamental component of our scheme, can be a source of decoherence *via* dissipation and noise. This means that a significant experimental challenge is the realization of a careful circuit design, balancing the competing demands for coupling and decoherence. Many authors are currently working in this field and their experimental results are encouraging in view of building a solid-state quantum computer using these macroscopic Josephson devices [23,35,36]. For example, measurements of the effective dissipation in *rf*-SQUID systems with features compatible with the conditions characterizing the strong coupling regime performed by Silvestrini's group at Naples have shown that it is possible to obtain very high quality factor ($Q \approx 10^5$ at $T = 2.3$ K) and then very low dissipation level [35]. These measurements are in agreement with the decoherence time of the order of 1 μs experimentally estimated by Cosmelli *et al.* for a system cooled at 5 mK and effective resistance $R \approx 4-5$ mK [36]. This means that we may think that this kind of experiment will be in the grasp of experimentalist in the next few years. In summary, we have theoretically studied the quantum mechanical dynamics of a field-qubit system in the strong coupling limit. We have demonstrated that, according to the current experimental techniques, it is possible to choose the system parameters in order to satisfy the conditions expressed by equation (8). Our results show that the irradiating electromagnetic field can be employed as an external tool for the control of the dynamics of the qubit. In particular, it is worth noting that the possibility to freeze the qubit in $|L\rangle$ or $|R\rangle$ state, tuning the system parameters to the case in which $\Delta' = 0$, is important in view of the realization of an electromagnetically driven superconducting quantum computer.

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